

GLUEBALL CORRELATORS AS HOLOGRAMS

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We investigate the dynamical content of both hard- and soft-wall approximations to holographic QCD by deriving the corresponding glueball correlation functions and by confronting them with a variety of QCD results. We further calculate the glueball decay constants in both holographic duals, discuss emerging limitations and improvement strategies, and comment on a recent attempt to generalize the glueball correlator in the soft-wall background.

1. Introduction

The discovery and ongoing development of gauge/string dualities¹ has opened up a new and exciting frontier for nonperturbative QCD. Already the current, first generation of “holographic QCD” or “AdS/QCD” duals with their bold approximations is beginning to provide new and often surprising analytical insights into the elusive infrared (IR) sector of the strong interactions.

By now a rather large set of static hadron properties has been calculated in the AdS/QCD framework (for recent reviews see e.g. Ref. [2]). The majority of this work was based on the two currently most popular dual candidates, i.e. the hard-³ and soft-wall⁴ backgrounds. Since even these rather minimal gravity backgrounds turn out to describe most calculated static hadron properties at an astonishing 10-30% accuracy level, it becomes increasingly important to explore their capacity and limitations in describing more detailed and sensitive QCD amplitudes. One such class of amplitudes comprises the hadron form-factors, and several of them have already been estimated holographically.⁵ Another important set of hadron amplitudes are the n -point functions of hadronic interpolating fields, and

among these the two-point correlators play a special role, not least because detailed QCD results are available for most of them.

We have therefore recently advocated⁶ to put AdS/QCD dual candidates to more stringent tests by evaluating their predictions for hadron correlators and by confronting those with QCD information from the lattice,⁷ the operator product expansion (OPE) including hard instanton contributions to the Wilson coefficients,⁸ a hypothetical UV gluon mass suggested to encode the short-distance behavior of the static quark-antiquark potential,⁹ and a scaling low-energy theorem¹⁰ based on the trace anomaly. (AdS/QCD correlators were recently also studied in Refs. [11–13].) In the following we will outline the main steps of implementing this program in the scalar glueball channel.⁶ To this end, we review the calculation of the scalar glueball spectra, decay constants and correlators in both hard- and soft-wall backgrounds, and we comment on a recent attempt¹² to generalize the soft-wall correlator.

2. Glueball spectra and decay constants

The holographic hard- and dilaton soft-wall duals are both based on five-dimensional bulk geometries of “Poincaré domain wall” type

$$ds^2 = g_{MN}(x) dx^M dx^N = e^{2A(z)} \frac{R^2}{z^2} (\eta_{\mu\nu} dx^\mu dx^\nu - dz^2) \quad (1)$$

where $\eta_{\mu\nu}$ is the four-dimensional Minkowski metric and conformal invariance of the dual gauge theory in the UV requires $A(z) \xrightarrow{z \rightarrow 0} 0$. The soft wall additionally contains a bulk dilaton field $\Phi(z)$.

Scalar QCD glueballs are interpolated by the lowest-dimensional gluonic operator carrying vacuum quantum numbers,

$$\mathcal{O}_S(x) = G_{\mu\nu}^a(x) G^{a,\mu\nu}(x) \quad (2)$$

(where $G_{\mu\nu}^a$ is the gluon field strength). Since \mathcal{O}_S has conformal dimension $\Delta = 4$ (at the classical level), the AdS/CFT dictionary¹ prescribes its dual string modes $\varphi(x, z)$ to be the normalizable solutions of the scalar wave equation in the bulk geometry (1) (and possibly other background fields) with the UV behavior $\varphi(x, z) \xrightarrow{z \rightarrow 0} z^\Delta \phi(x)$. The latter implies that the square mass $m_5^2 R^2 = \Delta(\Delta - d)$ of the bulk field φ vanishes for $d = 4$, so that its minimal action takes the form

$$S[\varphi; g, \Phi] = \frac{1}{2\kappa^2} \int d^5x \sqrt{|g|} e^{-\Phi} g^{MN} \partial_M \varphi \partial_N \varphi. \quad (3)$$

The four-dimensional Fourier transform $\hat{\varphi}(q, z)$ of the normalizable dual modes solves the reduced field equation

$$[\partial_z^2 + (d-1)(a^{-1}\partial_z a)\partial_z - (\partial_z \Phi)\partial_z + q^2]\hat{\varphi}(q, z) = 0 \quad (4)$$

obtained by variation of the bulk action (3). The corresponding orthonormalized solutions $\psi_n(z) = N_n \hat{\varphi}(m_n, z)$ have discrete momenta $q^2 = m_n^2$ which determine the glueball mass spectrum of the boundary gauge theory.

For dilaton fields which vanish at the UV boundary (as in the soft wall case), the glueball correlator has the spectral representation

$$\begin{aligned} \hat{\Pi}(-q^2) &= -i \int \frac{d^4 q}{(2\pi)^4} e^{iq(x-y)} \langle T \mathcal{O}_S(x) \mathcal{O}_S(y) \rangle \\ &= - \left(\frac{R^3}{\kappa \varepsilon^3} \right)^2 \sum_n \frac{\psi'_n(\varepsilon) \psi'_n(\varepsilon)}{q^2 - m_n^2 + i\varepsilon} = - \sum_n \frac{f_n^2 m_n^4}{q^2 - m_n^2 + i\varepsilon} \end{aligned} \quad (5)$$

where a prime denotes differentiation with respect to z and regularizing contact terms for $\varepsilon \rightarrow 0$ are not written explicitly. The pole residues of Eq. (5) contain the decay constants

$$f_n := \frac{1}{m_n^2} \langle 0 | \mathcal{O}_S(0) | 0_n^{++} \rangle = \frac{R^3}{\kappa m_n^2} \frac{\psi'_n(\varepsilon)}{\varepsilon^3} \quad (6)$$

of the n -th 0^{++} glueball excitation. Since the f_n can be regarded as the glueball (Bethe-Salpeter) wave functions at the origin, a smaller glueball size implies a higher concentration of the wave function and consequently a larger value of f_n . Evidence for such an enhancement of the ground-state decay constant was found in instanton vacuum models,¹⁴ in QCD sum rule analyses which include instanton contributions to the OPE coefficients,⁸ and in (quenched) lattice simulations.⁷

2.1. Hard wall

The hard-wall geometry of Polchinski and Strassler³ is an AdS_5 slice with a Randall-Sundrum type cutoff z_m at the IR brane,

$$e^{2A^{(\text{hw})}(z)} = \theta(z_m - z), \quad z_m \simeq \Lambda_{\text{QCD}}^{-1}, \quad \Phi^{(\text{hw})} \equiv 0, \quad (7)$$

which implements conformal symmetry in the UV and its breaking in the IR in a minimal fashion.

The glueball decay constants in the hard-wall background can be calculated directly from the normalized solutions

$$\psi_n(z) = N_n (m_n z)^2 J_2(m_n z) \quad (8)$$

(where $n = 1, 2, \dots$) of the field equation (4) in the metric (7). The constants N_n are determined by the normalization condition $\int_0^{z_m} dz (R/z)^3 \psi_n^2 = 1$. For the Dirichlet IR boundary condition $\psi_n(z_m) = 0$ one then obtains¹⁵

$$m_n^{(D)} = \frac{j_{2,n}}{z_m}, \quad N_n^{(D)} = \frac{\sqrt{2}}{m_n^{(D)2} R^{3/2} z_m |J_1(j_{2,n})|} \quad (9)$$

while the alternative Neumann boundary condition $\psi'_n(z_m) = 0$ yields¹⁵

$$m_n^{(N)} = \frac{j_{1,n}}{z_m}, \quad N_n^{(N)} = \frac{\sqrt{2}}{m_n^{(N)2} R^{3/2} z_m |J_0(j_{1,n})|}. \quad (10)$$

Here $j_{m,n}$ denotes the n -th zero of the m -th Bessel function.¹⁶

From the general expression (6) for the decay constants and the hard-wall eigenmodes (8) one then finds⁶

$$f_n = \lim_{\varepsilon \rightarrow 0} \frac{R^3}{\kappa m_n^2} \frac{\psi'_n(\varepsilon)}{\varepsilon^3} = \frac{N_n}{2} \frac{R^3}{\kappa} m_n^2 \quad (11)$$

or more specifically for the above two IR boundary conditions

$$f_n^{(D)} = \frac{1}{\sqrt{2} |J_1(j_{2,n})|} \frac{R^{3/2}}{\kappa z_m}, \quad f_n^{(N)} = \frac{1}{\sqrt{2} |J_0(j_{1,n})|} \frac{R^{3/2}}{\kappa z_m}. \quad (12)$$

For a quantitative estimate one can fix the overall normalization factor $R^{3/2}/\kappa$ according to Eq. (23) and set the IR scale $z_m^{-1} \sim \Lambda_{\text{QCD}}$ e.g. such that a typical quenched ground-state glueball mass of $m_S \sim 1.5$ GeV is reproduced, or at the value $z_m^{-1} \simeq 0.35$ GeV found in the classical hadron sector. Either way, the ground-state decay constant predictions remain in the range $f_S^{(\text{hw})} \equiv f_1^{(\text{hw})} \simeq 0.8 - 0.9$ GeV.⁶

2.2. Soft wall

The hard-wall predictions (9), (10) for the squared masses of scalar glueballs (and of other hadrons) grow quadratically with high radial (and orbital) excitation quantum numbers, in contrast to the linear trajectories expected from semiclassical models and data. The dilaton soft-wall background⁴

$$A^{(\text{sw})}(z) \equiv 0, \quad \Phi^{(\text{sw})}(z) = \lambda^2 z^2 \quad (13)$$

provides an economical corrective to this problem in the meson⁴ and glueball¹⁷ sectors. (The “metric soft wall”¹⁸ is a dilaton-less alternative which also yields linear baryon mass trajectories.)

The spectrum-generating normalizable solutions of Eq. (4) in the background (13) are those Kummer functions whose power series expansion truncates to generalized Laguerre polynomials $L_n^{(2)}$,¹⁶ i.e.

$$\psi_n(z) = N_n \lambda^4 z^4 {}_1F_1(-n, 3, z^2 \lambda^2) = N_n \lambda^4 z^4 \frac{n!}{(3)_n} L_n^{(2)}(\lambda^2 z^2) \quad (14)$$

where $n = 0, 1, 2, \dots$, $(a)_n \equiv a(a+1) \dots (a+n-1)$ and ${}_1F_1$ is a confluent hypergeometric function.¹⁶ The ensuing restriction to discrete $q^2 = m_n^2$ generates the mass gap $m_0 = 2\sqrt{2}\lambda$ and the glueball mass spectrum¹⁷

$$m_n^2 = 4(n+2)\lambda^2 \quad (15)$$

which indeed lies on a linear Pomeron-type trajectory. The condition $\int_0^\infty dz (R/z)^3 \exp(-\lambda^2 z^2) \psi_n^2(z) = 1$ fixes the normalization constants

$$N_n = \lambda^{-1} R^{-3/2} \sqrt{\frac{(n+1)(n+2)}{2}} \xrightarrow{n \gg 3} 2^{-1/2} \lambda^{-1} R^{-3/2} n. \quad (16)$$

From the general expression (6) one then finds the glueball decay constants in the soft-wall background as⁶

$$f_n = 2\sqrt{2(n+1)(n+2)} \frac{\lambda^3 R^{3/2}}{m_n^2 \kappa} = \frac{1}{\sqrt{2}} \sqrt{\frac{n+1}{n+2}} \frac{\lambda R^{3/2}}{\kappa}. \quad (17)$$

After fixing the factor $R^{3/2}/\kappa$ by Eq. (23) one can as above obtain quantitative estimates for the f_n by setting the IR scale λ either such as to reproduce the typical quenched mass value $m_S \sim 1.5$ GeV or by using the value $\lambda \simeq \sqrt{2}\Lambda_{\text{QCD}} \simeq 0.49$ GeV of Ref. [18]. Both variants lead to similar soft-wall predictions $f_S^{(\text{sw})} \equiv f_0^{(\text{sw})} \simeq 0.3$ GeV for the ground state decay constant.⁶

3. Holographic glueball correlators

Holographic correlation functions can be derived from the on-shell action of the gravity dual which plays the role of their generating functional. To construct it, one employs the bulk-to-boundary propagator $\hat{K}(q, z)$,¹⁹ i.e. the solution of the field equation (4) subject to the $\hat{K}(q; \varepsilon \rightarrow 0) = 1$ and $\hat{K}(0; z) = 1$ boundary conditions, to write the solution of Eq. (4) with a boundary source $\varphi^{(s)}(x')$ as

$$\varphi(x, z) = \int \frac{d^4 q}{(2\pi)^4} e^{-iqx} \hat{K}(q, z) \int d^4 x' e^{iqx'} \varphi^{(s)}(x'). \quad (18)$$

Inserting the solution (18) into Eq. (3) yields the on-shell action, and taking two functional derivatives with respect to $\varphi^{(s)}$ then generates the correlator

$$\hat{\Pi}(-q^2) = -\frac{R^3}{\kappa^2} \left[\frac{e^{-\Phi(z)}}{z^3} \hat{K}(q, z) \partial_z \hat{K}(q, z) \right]_{z=\varepsilon \rightarrow 0} \quad (19)$$

of the scalar glueball. Analytical solutions for $\hat{K}(q, z)$ in both hard- and soft-wall backgrounds were found in Ref. [6].

3.1. Hard wall

After plugging the analytical hard-wall solution for $\hat{K}(q, z)$ ⁶ (subject to the Neumann IR boundary condition) into the general expression (19) and discarding two contact terms, one ends up with the correlator⁶

$$\hat{\Pi}(Q^2) = \frac{R^3}{8\kappa^2} Q^4 \left[2 \frac{K_1(Qz_m)}{I_1(Qz_m)} - \ln \left(\frac{Q^2}{\mu^2} \right) \right] \quad (20)$$

(K_ν, I_ν are McDonald functions¹⁶) at spacelike momenta $Q^2 = -q^2$. Its spectral density $\rho(s)$ is defined by means of the dispersion relation

$$\hat{\Pi}(Q^2) = \int_{m_{\min}^2}^{\infty} ds \frac{\rho(s)}{s + Q^2} \quad (21)$$

(suppressing again subtraction terms) and takes the form⁶

$$\rho(s) = \frac{R^3}{2\kappa^2 z_m^2} s^2 \sum_{n=1}^{\infty} \frac{\delta(s - m_n^2)}{J_0^2(j_{1,n})} \quad (22)$$

where the hard-wall mass spectrum $m_n = j_{1,n}/z_m$ reappears. The spectral weight (22) is non-negative and consists of zero-width poles, as expected in the large- N_c limit where glueballs are stable against strong decay.

The overall correlator normalization R^3/κ^2 is fixed by matching the leading conformal logarithm to the free QCD gluon loop,

$$\frac{R^3}{\kappa^2} = \frac{2(N_c^2 - 1)}{\pi^2} \quad (23)$$

with $N_c = 3$. For $Q \gg \mu > z_m^{-1}$ the holographic correlator (20) can be compared to the QCD short-distance expansion.⁸ The exponential Q^2 dependence (times powers of Q^2) of its non-conformal part

$$\hat{\Pi}^{(\text{np})}(Q^2) \xrightarrow{Qz_m \gg 1} \frac{4}{\pi} \left[1 + \frac{3}{4} \frac{1}{Qz_m} + O\left(\frac{1}{(Qz_m)^2}\right) \right] Q^4 e^{-2Qz_m} \quad (24)$$

has its QCD OPE counterpart in the small-size instanton contribution⁸

$$\hat{\Pi}^{(I+\bar{I})}(Q^2) \xrightarrow{Q\bar{\rho} \gg 1} 2^4 5^2 \pi \zeta \bar{n} (Q\bar{\rho})^3 e^{-2Q\bar{\rho}} \quad (25)$$

to the unit operator coefficient. Since the instanton-induced correlations are attractive and of relatively short range $\sim \bar{\rho}$, they reduce the scalar glueball mass and size while increasing its decay constant.⁸

Approximately equating Eq. (25) to the second term in Eq. (24) yields the holographic estimates

$$\bar{\rho} \simeq z_m, \quad \bar{n} \simeq \frac{3}{2^4 5^2 \pi^2 \zeta} \frac{1}{z_m^4} \quad (26)$$

for the average instanton size $\bar{\rho}$ and the overall instanton density \bar{n} in terms of the IR scale z_m . The first relation reflects the duality between gauge instantons of size ρ and pointlike D instantons localized at $z = \rho$. With $z_m^{-1} \sim \Lambda_{\text{QCD}} \simeq 0.33$ GeV it implies $\bar{\rho} \sim 0.6$ fm, i.e. almost twice the standard value.¹⁴ This may suggest that the bulk dynamics (3) is more suitable for pure Yang-Mills theory with its larger instanton sizes $\bar{\rho} \simeq 0.4 - 0.5$ fm, and that the strongly coupled hard-wall UV dynamics describes the small-instanton physics beyond the conformal regime rather poorly.

The above discussion implies that $\rho^{(\text{hw})} \leq z_m \sim \mu^{-1}$, i.e. that large instantons, which would contribute to the condensates, are absent since their duals do not fit into the AdS₅ slice. Hence the hard wall reproduces the QCD result that the *hard* nonperturbative physics from small instantons (instead of the soft condensate physics) dominates the short-distance 0^{++} glueball correlator.⁸ Moreover, the QCD low-energy theorem¹⁰

$$\hat{\Pi}(0) = \frac{32\pi}{\alpha_s b_0} \langle G^2 \rangle + O(m_q) \quad (27)$$

with $b_0 = 11N_c/3 - 2N_f/3$ is trivially satisfied by the hard-wall correlator (20) which vanishes at $Q^2 = 0$. This is consistent with Eq. (27) since the gluon condensate vanishes in the hard wall as well.

3.2. Soft wall

Inserting the analytical solution for the soft-wall bulk-to-boundary propagator $\hat{K}(q; z)$ ⁶ into Eq. (19) yields (after discarding two divergent contact terms) the soft-wall correlator⁶

$$\hat{\Pi}(Q^2) = -\frac{2R^3}{\kappa^2} \lambda^4 \left[1 + \frac{Q^2}{4\lambda^2} \left(1 + \frac{Q^2}{4\lambda^2} \right) \psi \left(\frac{Q^2}{4\lambda^2} \right) \right] \quad (28)$$

in terms of the digamma function $\psi(z) = \Gamma'(z)/\Gamma(z)$.¹⁶ The analyticity structure of Eq. (28) implies that its spectral density has the form⁶

$$\rho(s) = \frac{\lambda^2 R^3}{2\kappa^2} s(s - m_0^2/2) \sum_{n=0}^{\infty} \delta(s - m_n^2) = \sum_{n=0}^{\infty} f_n^2 m_n^4 \delta(s - m_n^2) \quad (29)$$

which is non-negative for $s \geq m_0^2/2$ and consists of zero-width poles, as expected at large N_c , at the soft-wall masses (15) with residua determined by the soft-wall decay constants (17).

In order to compare the soft-wall correlator (28) to the QCD OPE, we use the asymptotic expansion of the digamma function to rewrite Eq. (28) for $Q^2 \gg 4\lambda^2 > \Lambda_{\text{QCD}}^2$ as

$$\hat{\Pi}(Q^2) = -\frac{2}{\pi^2} Q^4 \left[\ln \frac{Q^2}{\mu^2} + \frac{4\lambda^2}{Q^2} \ln \frac{Q^2}{\mu^2} + \frac{2^2 5}{3} \frac{\lambda^4}{Q^4} - \frac{2^4}{3} \frac{\lambda^6}{Q^6} + \frac{2^5}{15} \frac{\lambda^8}{Q^8} + \dots \right] \quad (30)$$

which is renormalized at the OPE scale μ . The normalization R^3/κ^2 is again fixed by Eq. (23) since large momenta Q probe the $z \rightarrow 0$ region where hard- and soft-wall correlators are governed by the same AdS_5 -induced logarithm.

Besides the leading conformal and a second logarithmic term, the expansion (30) contains an infinite tower of power corrections. Comparison with the OPE suggests those to be related to the gauge-theory condensates $\langle \mathcal{O}_D \rangle \sim \lambda^D$ of $D = 4, 6, 8, \dots$ dimensional composite operators. For a first order-of-magnitude check one may equate the coefficients of the $D = 4, 6$ and 8 terms in Eq. (30) to the $(O(\alpha_s^0))$ QCD Wilson coefficients, yielding

$$\langle G^2 \rangle \simeq -\frac{10}{3\pi^2} \lambda^4, \quad \langle gG^3 \rangle \simeq \frac{4}{3\pi^2} \lambda^6, \quad \langle G^4 \rangle \simeq -\frac{8}{15\pi^3 \alpha_s} \lambda^8. \quad (31)$$

For $\lambda \sim \Lambda_{\text{QCD}}$ these are the rough magnitudes of the QCD condensates, but the sign of the QCD gluon condensate $\langle G^2 \rangle \sim 0.4 - 1.2 \text{ GeV}^4$ is positive. While QCD estimates of both signs exist for the three-gluon condensate, the above signs would also be at odds with the factorization approximation $\langle G^4 \rangle \simeq (9/16) \langle G^2 \rangle^2$.

The probably most intriguing prediction of the soft-wall correlator (30) is the additional power correction of dimension two which cannot appear in the OPE since QCD lacks a corresponding local operator. When linear contributions to the *short-distance* heavy-quark potential are approximately described by a tachyonic gluon mass $\bar{\lambda}$, however, one finds the correction⁹

$$\hat{\Pi}_{\bar{\lambda}}^{(\text{CNZ})}(Q^2) = -\frac{12}{\pi^2} \bar{\lambda}^2 Q^2 \ln \frac{Q^2}{\mu^2} \quad (32)$$

which has precisely the form of the second term in Eq. (30). Comparison of the coefficients provides the holographic estimate $\bar{\lambda}^2 \simeq (2/3)\lambda^2$ and with $\lambda \simeq \sqrt{2}\Lambda_{\text{QCD}}$ further $\bar{\lambda}^2 \simeq 0.15 \text{ GeV}^2$ which is of the expected magnitude⁹ but again of opposite sign, i.e. not tachyonic.

Hence the expansion (30) completely reproduces the qualitative Q^2 dependence of the QCD short-distance correlator (to leading order in α_s) but fails to predict the sign of at least the two leading terms. This pattern can be better understood by recalling that the dimensions of the QCD condensates are generated by operators which are renormalized at $\mu \lesssim 1 \text{ GeV}$ and thus IR dominated. The form and general Q^2 dependence of the QCD power corrections is therefore governed by IR physics, which may explain why the strongly-coupled soft-wall dynamics can reproduce it. The deviations of size and signs of the holographic power corrections from their QCD counterparts (and the absence of radiative corrections) should then originate mainly from the poorer description of the weakly-coupled, perturbative Wilson coefficients by the soft-wall dynamics which lacks α' corrections and remains strongly coupled in the UV.

The above interpretation also suggests an approximate separation of the soft-wall power corrections into Wilson coefficients and condensates. Indeed, under the premise that the strongly-coupled soft wall dynamics approximately reproduces the QCD condensate values, one may obtain holographic estimates for the Wilson coefficients. The gluon condensate coefficient, e.g., becomes with $\langle G^2 \rangle \simeq (20/3)\Lambda_{\text{QCD}}^4$ ⁸ and $\lambda \simeq \sqrt{2}\Lambda_{\text{QCD}}$ ¹⁸

$$C_{\langle G^2 \rangle}^{(\text{sw})} \simeq -\frac{8}{\pi^2} = -\frac{2}{\pi^2} C_{\langle G^2 \rangle}^{(\text{QCD}, \text{lo})}. \quad (33)$$

Its smaller size and opposite sign relative to the QCD result provides some intuition for the soft-wall deficiencies in describing weakly-coupled QCD physics. Of course, the estimate (33) is prone to further error sources, including uncertainties in the QCD condensate values and their sensitivity to light quark contributions. (The above separation into hard and soft contributions would fail for the two-dimensional power correction, incidentally, since both the gluon mass $\bar{\lambda}$ and its coefficient receive UV contributions.) Since the condensates are hadron-channel independent while the Wilson coefficients are not, one would further expect to obtain inconsistent condensate estimates when trying to extract them in different channels by relying on the respective QCD Wilson coefficients. Finally, we note that the soft-wall correlator (28) with $\hat{\Pi}(0) = 0$ violates the low-energy theorem (27) since Eq. (31) implies a finite RHS.

3.3. A generalized soft-wall correlator?

Recently an attempt has been made to generalize the soft-wall glueball correlator (28) by adding to the bulk-to-boundary propagator $\hat{K}(q, z)$ the at small z subleading solution of Eq. (4), multiplied by an a priori arbitrary coefficient function $\tilde{B}(Q^2)$.¹² The added solution blows up at large z , i.e. it violates the standard “regularity in the bulk” condition¹⁹ and requires an ad-hoc IR cutoff prescription without obvious correspondence on the gauge-theory side (in contrast to the standard UV renormalization whose “dual” tames the volume divergence of the on-shell bulk action). The resulting expression for the correlator differs from ours, i.e. Eq. (28), by the addition of the arbitrary coefficient function $\tilde{B}(Q^2)$. Any desired behavior of the correlator could thus be chosen by hand, and independently of the soft-wall background, by adapting $\tilde{B}(Q^2)$ accordingly. (Ref. [12] attempted to use this apparent freedom to equate the $D \geq 4$ power corrections to their QCD values and to eliminate the two-dimensional power correction.)

Hence the prescription of Ref. [12] gives up on the one-to-one duality between the gauge vacuum and the gravity background (together with other parts of the AdS/CFT dictionary) and results in a practically total loss of predictive power. Moreover, it is internally inconsistent. Indeed, it was shown to result in the same mass spectrum (15) and the same decay constants (17) as in our case.¹² Hence it must reproduce our spectral density (29) and thus the physics content of the correlator (28), i.e. any remaining discrepancies have to originate from the unphysical contact terms which are needed to regularize the dispersion integral (21). This is in direct contradiction to the supposed freedom of adding an arbitrary function $\tilde{B}(Q^2)$ to the correlator.

The loss of predictivity, uniqueness and consistency incurred when relaxing the regularity of $\hat{K}(q, z)$ in the bulk has a common origin. Indeed, any given (smooth) function defined on the UV boundary S^d of (Euclidean and compactified) AdS_{d+1} is known to have a *unique* extension to a solution of the massless scalar field equation in the AdS_{d+1} bulk.¹⁹ When applied to the boundary source $\varphi^{(s)}(x)$, this mathematical fact ensures the one-to-one correspondence between the gauge-theory operators and the dual string mode solutions. The inconsistency of the attempted generalization of the bulk-to-boundary propagator arises from the violation of this fact. In order to maintain the mathematically required uniqueness of the relation (18) between a given boundary source $\varphi^{(s)}(x)$ and its dual mode solution $\varphi(x, z)$, the UV-subleading solution must not be added to $\hat{K}(q, z)$.

4. Summary and conclusions

We have derived and analyzed the predictions of the two currently most popular AdS/QCD duals, i.e. the hard-wall and dilaton soft-wall backgrounds, for the 0^{++} glueball correlation function and decay constants.

In their representation of specific nonperturbative glueball physics (at momenta larger than the QCD scale) both holographic duals turn out to complement each other: the soft-wall correlator contains all known types of QCD power corrections, generated both by vacuum condensates and by a hypothetical UV gluon mass, while sizeable exponential corrections of the type induced by small-scale QCD instantons are reproduced in the hard-wall correlator. Since the QCD power corrections to the 0^{++} glueball correlator are suppressed by unusually small Wilson coefficients whereas the small-instanton contributions are enhanced, the hard-wall background may provide a more reliable approximation to the scalar glueball correlator. Furthermore, the above complementarity, which helps to relate holographic predictions (and their limitations) to specific aspects of the gauge dynamics, should extend to other hadron channels.

While all our holographic estimates have the order of magnitude expected from QCD, the signs of the two leading soft-wall power corrections are opposite to those of standard QCD estimates (and in conflict with the factorization approximation for the four-gluon condensate). We have argued that this provides evidence for the short-distance physics in the Wilson coefficients to be inadequately reproduced by the strongly-coupled UV dynamics of bottom-up models (beyond the leading conformal logarithm). We have further shown that this problem cannot be mended by admixing the UV-subleading solution to the bulk-to-boundary propagator (as recently advocated) without losing consistency and predictive power.

In addition, we have provided first holographic estimates for the 0^{++} glueball decay constants which contain glueball size information, are important for experimental glueball searches and probe aspects of the dual dynamics to which the mass spectrum is less sensitive. The hard-wall prediction for the ground-state decay constant f_S is more than twice as large as its soft-wall counterpart. This is a consequence of the exponential contributions to the hard-wall correlator which reproduce the strong instanton-induced short-distance attraction in the QCD correlator. The hard-wall prediction $f_S^{(\text{hw})} \simeq 0.8 - 0.9 \text{ GeV}$ implies an exceptionally small glueball size and agrees inside errors with IOPE sum-rule and lattice results.

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